Mathematical Proofs

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Abstract

"The meaning of a proof."
"How to make the internal structure of a proof transparent."
"Suppose we want to prove theorem A:
Proofs via inner structures: The main body is the theorem A. The proof uses all the available resources within the hypothesis of the thorem and the existence is a construction without resorting to outer theories. If computation keeps refined by the same method, the solution can be approximated with an arbitrarily prescribed accuracy.
Proofs via outer structures: The main body is an outer theory. Theorem A is an object used to compare with theorems in the outer theory. The proof uses reduction to absurdity. The negation of Theorem A will lead to a contradiction to theorem B in the outer theory. There is no way to approximate solutions."

Keywords.

1 The meaning of a proof

The meaning of a theorem's proof is the interpretation of the theorem. The simpler and clearer the interpretation is, the better the proof is. The more easily a proof can be understood, the better the proof is. The fewer tools and settings a proof uses, the better the proof is. A good proof will make its internal structure transparent. That is, there are no black box operations; we know what we are doing in the entire proving process.

2 How to make the internal structure of a proof transparent

Blaga [1, §1.14] proves existence and uniqueness theorems for parameterized curves. In Blaga [1, p.68, Definition 1.14.1], $\mathcal{A}$ is defined as an orthogonal matrix with determinant equal to one. Blaga [1, p.69, l.1] claims that $\mathcal{A}$ is a rotation, but fails to provide a proof. In order to prevent Blaga [1, §1.14] from becoming black box operations, Blaga should have provided a solution of O'neill [2, p.111, Exercise 4] before the claim. Similarly, in order to prevent O'neill [2, chap. III; especially, pp.117–118, the proof of Theorem 5.3] from becoming black box operations, O'neill should have placed the solution of O'neill [2, p.111, Exercise

4] immediately after O'neill [2, p.100, l.13] as a lemma.
Remark. An intrinsic proof does not necessarily mean a natural proof: O'neill [2, p.331, l.−17–l.−15].

3 Proofs via inner structures vs. proofs via outer sructures

Suppose we want to prove theorem A: the fundamental theorem of algebra.
Proofs via inner structures [Uspensky [4, p.293, the fundamental theorem of algebra]]: The main body is the theorem A. The proof uses all the available resources within the hypothesis of the thorem and the existence is a construction without resorting to outer theories. If computation keeps refined by the same method, the solution can be approximated with an arbitrarily prescribed accuracy.
Proofs via outer structures [van der Waerden [5, vol.1, p.223, the fundamental theorem of algebra]; Rudin [3, p.229, Theorem 10.25]]: The main body is an outer theory. Theorem A is an object used to compare with theorems in the outer theory. The proof uses reduction to absurdity. The negation of Theorem A will lead to a contradiction to theorem B [Rudin [3, p.228, Theorem 10.23; p.229, Theorem 10.24]] in the outer theory [Complex Variables]. There is no way to approximate solutions.

References

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